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ITERATIVE INFORMATION-REDUCED CARRIER SYNCHRONIZATION USING DECISION FEEDBACK FOR LOW SNR APPLICATIONS[1]

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ABSTRACT

Traditional methods for carrier synchronization of digital signals are obtained from approximations made to a closed loop structure motivated by the maximum a posteriori (MAP) estimation of carrier phase. Inherent in all of these loops is the fact that their input data is assumed to be an equiprobable (balanced) independent identically distributed (i.i.d.) binary sequence, and the MAP estimation loop from which these various structures are derived is predicated on this fact. By *reducing* the amount of randomness (information) in the data that is *input* to the carrier synchronizer (hence the term *information-reduced carrier synchronization*), yet maintaining its i.i.d (but not necessarily balanced) property and also its independence of the additive noise (both of which can be satisfied for coded modulations), and then suitably modifying the synchronizer structure in accordance with this data reduction, one can obtain a significant loop SNR improvement relative to what is achievable with the above-mentioned structures. Although the paper focusses on binary modulation, the concepts and results are easily extended to higher order M-PSK modulations with $M \geq 4$.

INTRODUCTION

Optimum closed loop structures for tracking the phase of BPSK signals as well as unmodulated carriers have been derived in the past based on maximum-a-posteriori (MAP) estimation criteria [1-3]. These structures have been shown to take the form of feedback loops which attempt to null the difference between the true input phase and its estimate produced by an oscillator in the loop. It has been customary in the past to distinguish between unmodulated and modulated carriers before deriving the optimum closed loop structures, the former leading to a phase-locked loop (PLL) and the latter leading to an inphase-quadrature (I-Q) Costas-type loop with a hyperbolic tangent nonlinearity in its inphase arm.

It is well known that PLLs attain a phase error variance that, in the limit of large loop SNR, varies as the inverse of this loop SNR whereas I-Q loops suffer an additional degradation because of the inherent multiplication required to produce an error signal in the I-Q structure. This multiplication of the I and Q signals results in the generation of signal and noise cross-products which leads to a loop SNR penalty (i.e., multiplication of the loop SNR by a factor less than unity) generically referred to as *squaring loss*. At low symbol SNR,[2] the squaring loss degradation associated with I-Q loops can be severe often prohibiting the ability to track. This suggests that substantial improvements may be possible if somehow one could convert the received modulated carrier to a pure (unmodulated) tone *before* applying it to the phase-tracking loop since this would then allow use of a PLL which, as mentioned above, has no associated squaring loss penalty.

In principle, an uncoded BPSK signal could indeed be converted to a pure tone if the data sequence were completely known simply by multiplying the BPSK signal by the data waveform. Short of complete knowledge of the data waveform and in the presence of noise, the next best attempt at arriving at a "pure" input tone would be to feed back decisions (estimates) made on the data symbols. Referring to Fig. 1, the above statements can be put into mathematical terms as follows. Denoting the received signal plus noise waveform by $x(t;a_k,\theta)$ where a_k is the kth data symbol taking on values ± 1 with equal probability and θ is the unknown carrier phase to be estimated, it follows that if a perfect estimate, $\hat{a}_k$, of the data were available, so that $\hat{a}_k = a_k$, then the product of a_k and $\hat{a}_k$ would always be unity and the data-modulation would in effect be removed from the signal input to the carrier synchronizer. Implicit in the previous statement is the assumption that accurate symbol synchronization has already been established and that data estimation delays have been accounted for by means of the signal delay Δ prior to the multiplication operation. If perfect symbol estimates are not available, the product of a_k and $\hat{a}_k$ forms an error sequence $e_k \triangleq a_k \hat{a}_k$ with statistics based on the probability p assigned to the error event $\hat{a}_k \neq a_k$. Thus, ignoring for the moment the effect of this data estimate

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[2] We use the term "symbol" rather than "bit" to include the case of coded data which will be our main focus of interest. Furthermore, it should be noted that assumption of low *symbol* SNR does not necessarily invalidate the assumption of high *loop* SNR.

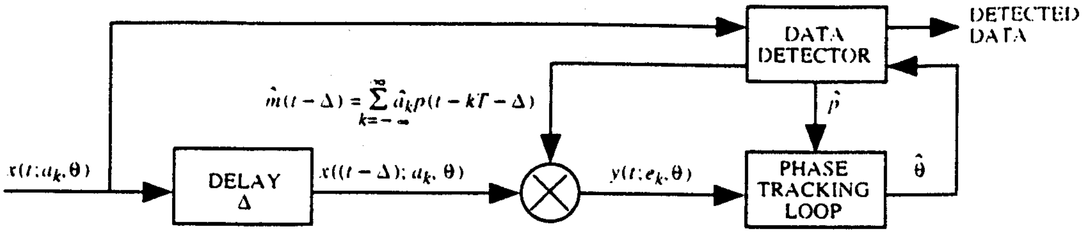

Fig. 1. Receiver with information-reduced carrier phase estimation.

multiplication on the noise component of $x(t;a_k,\theta)$ (which as we shall see shortly is an important consideration and is, in fact, what prevents this scheme from being successful for uncoded modulations), the input to the loop has been transformed into $y(t;e_k,\theta)$ whose signal component now represents the carrier modulated by the error sequence instead of the original data sequence. Assuming that the data estimates are i.i.d. (as would be the case for uncoded data), then the error sequence $\{e_k\}$ is an unbalanced i.i.d sequence with the data estimate error probabability p now also representing the probability that e_k takes on a value equal to minus one. Hence a value of $p=.5$ results in a balanced i.i.d. sequence and thus the feedback provides no advantage whereas a reduction in the value of p to $p=0$ would result in an all ones input sequence, i.e., an unmodulated carrier. The degree to which p can be reduced from its nominal value of $p=.5$ depends on the nature of the data estimator (detector) which for uncoded data would be a matched filter followed by a hard decision threshold device.

If it were not for the effect of the decision feedback on the noise component of the received signal which is input to the loop, then modification of the loop structure in accordance with MAP theory based on the changed data sequence statistics discussed above should yield an improvement in performance. Again this would occur because any unbalance (tilt) in the received data statistics, i.e., p other than 0.5, brings the signal component of the received signal closer to a "pure" tone which should intuitively allow phase estimation with a smaller squaring loss. Unfortunately, for bit-by-bit detection of uncoded BPSK with a matched filter, each data decision that is outputted from the hard decision threshold device is based on the same symbol interval of noise as the corresponding interval of noise in the received signal that gets multiplied by this decision. Stated another way the error statistic $e_k \triangleq a_k\hat{a}_k$ (which of course contains the decision $\hat{a}_k$) corresponding to the kth transmission interval is totally correlated with the noise component of $x(t;a_k,\theta)$ during that same time interval and thus the assumption of a modified data sequence input to the loop which is independent of the additive noise is invalid. Indeed for the configuration of Fig. 1 when implemented for uncoded BPSK,[3] because of the above-mentioned data sequence and noise correlation the squaring loss performance cannot be improved relative to the well-known conventional techniques. This has been formally proved in [7].

Suppose now that the BPSK symbols correspond to encoded data and the decisions result from, say, a convolutional (conventional or iterative) decoder. Then a memoryless data model for the error sequence at the phase tracking loop input is strictly speaking inappropriate. If the joint statistics of the input symbols were known as well as the actual decisions statistics, then, in principle, the structure of the MAP estimator could be derived. However, the mathematics would become intractable and thus it behooves us to find a way to justify the i.i.d. and noise independence assumptions for the error sequence. One simple way of validating the i.i.d assumption is to preprocess (e.g., interleave) the data input to the carrier synchronizer and at the same time perform the same preprocessing of the data being fed back from the decoder in such a way as to randomize the error sequence. We point out that such processing of the data and data feedback used as inputs to the carrier synchronizer does not affect the actual data decoding process since this is performed external to the carrier synchronization operation. To validate the assumption that the components of the error sequence are approximately independent of the additive noise in the corresponding transmission intervals (a situation that as mentioned above is not possible for the uncoded data case), we argue as follows. In a coded system, the decisions on the data information bits (or equivalently on the encoded data symbols as is needed for the carrier synchronizer) are made based on an observation interval of the received signal much longer than a single transmission interval. For example, in a convolutionally encoded data system, the output decision of the maximum-likelihood (ML) decoder, e.g, the Viterbi decoder, is obtained from the first bit of the current ML sequence whose length corresponds to a number of code constraint lengths sufficient to have all surviving paths converge at

[3] Note that in this case the delay Δ becomes equal to the bit time T.

their origin.[4] Thus, the noise that determines this ML sequence decision and hence the decision on the first bit (or equivalently the first encoded symbols) in this sequence is virtually uncorrelated with the noise in the same first transmission interval.

Assuming the above unbalanced i.i.d models, the next section presents the open loop MAP estimator of carrier phase and the closed loop carrier synchronizer motivated by this approach.

MAP ESTIMATION OF CARRIER PHASE FOR BPSK WITH UNBALANCED DATA

Consider the estimation of carrier phase for uncoded BPSK signals with unbalanced data in the presence of additive white Gaussian noise (AWGN). To begin, the received signal is of the form

$$s(t;\theta) = \sqrt{2S}m(t)\sin\left(\omega_c t + \theta(t)\right) \tag{1}$$

while the additive noise has the narrowband representation

$$n(t) = \sqrt{2}\left[N_c(t)\cos\left(\omega_c t + \theta(t)\right) - N_s(t)\sin\left(\omega_c t + \theta(t)\right)\right] \tag{2}$$

The carrier in (1) has power S, radian frequency ω_c, slowly varying phase $\theta(t)$ and is modulated by a random pulse train

$$m(t) = \sum_{k=-\infty}^{\infty} a_k p(t-kT) \tag{3}$$

where $p(t)$ is a unit power rectangular pulse of duration T sec and $\{a_k\}$ is a binary (±1) equiprobable data sequence assumed to be i.i.d. The noise process in (2) is modeled in terms of a pair of independent, low-pass Gaussian processes $N_c(t), N_s(t)$, each with single-sided power spectral density (PSD) N_0 Watts/Hertz and bandwidth $B < \omega_c / 2\pi$.

In accordance with Fig. 1, the received signal plus noise, $x(t;a_k,\theta)$, is delayed by an amount Δ and then multiplied by the data estimator waveform

$$\hat{m}(t-\Delta) = \sum_{k=-\infty}^{\infty} \hat{a}_k p(t-kT-\Delta) \tag{4}$$

where $\{\hat{a}_k\}$ denotes the sequence of binary (±1) data estimates. Assuming, as previously discussed, that this sequence of estimates is i.i.d. and has the probability statistics

$$\Pr\{\hat{a}_k \neq a_k\} = p,\ \Pr\{\hat{a}_k = a_k\} = 1-p \tag{5}$$

where p represents the error probability of the data estimator, then the result of the product of $x(t-\Delta)$ and $\hat{m}(t-\Delta)$ can be modeled as[5]

$$y(t) = \sqrt{2S}e(t)\sin\left(\omega_c t + \theta(t)\right) + N(t) \tag{6}$$

where

$$e(t) = \sum_{k=-\infty}^{\infty} e_k p(t-kT) \tag{7}$$

is a modified data modulation that reflects the errors in the detection of the input data sequence, i.e., $\{e_k\}$ is a binary (±1) i.i.d. sequence with probability statistics[6]

$$\Pr\{e_k = -1\} = p,\ \Pr\{e_k = 1\} = 1-p \tag{8}$$

and $N(t) \triangleq \hat{m}(t)n(t)$ is an additive noise process that is Gaussian in each T-sec interval with identical statistics to $n(t)$ and, based on the arguments previously given for the coded application, is independent of the data.

The combined signal plus noise in (6) is observed for K data intervals, i.e., over the interval $0 \le t \le KT$. Assuming that the carrier phase $\theta(t)$ remains constant over this observation interval, then based on this observation and knowledge of $S, p(t)$, and ω_c, the MAP estimate of phase is that value $\hat{\theta}_{MAP}$ that maximizes the conditional probability density function $p(\theta|y(t))$ or equivalently $p(y(t)|\theta)$ since $\theta(t) = \theta$ is assumed to be uniformly distributed. Since $\hat{\theta}_{MAP}$ is the value of θ that maximizes the log likelihood function, an equivalent statement is that $\hat{\theta}_{MAP}$ is the value of θ at which the derivative of the likelihood function has zero value (and the second derivative is negative). For estimates $\hat{\theta}$ of θ in the neighborhood of $\hat{\theta}_{MAP}$ this derivative will be positive or negative in accordance with the sign of $\hat{\theta} - \hat{\theta}_{MAP}$ and thus it can be used as an error signal in a closed loop synchronizer to steer the loop in the direction of a locked condition corresponding to $\hat{\theta} = \hat{\theta}_{MAP}$. Using the well-known MAP estimation approach, it is straightforward to show that for the case of unbalanced data, the derivative of the log likelihood function is given by [4,7]

[4] It is envisioned for such applications that the symbol decision feedback could be obtained by storing the encoded symbol history of the survivor paths associated with the decoding algorithm rather than the input history normally associated with these paths. This soft symbol decision information would then be fed back synonymous with the time at which one would normally start making bit decisions with the decoder in which case the delay Δ in Fig. 1 would be set equal to the decoder delay.

[5] In what follows we ignore the delay Δ (i.e., set it equal to zero) since its value has no bearing on the derivation that follows. Also, for simplicity of notation, we herein use $y(t)$ instead of $y(t;e_k,\theta)$.

[6] In reality, the error probability of the decisions being fed back will be dependent on the tracking performance of the carrier synchronization loop itself since indeed the phase of the inphase demodulation reference signal (the output of the phase tracking loop in Fig. 1) is an input to the data detector. We ignore this effect since for large loop SNR, which is the case of interest, this is a second order effect.

$$\frac{\partial}{\partial\theta}\ln\Lambda(\theta)=\sum_{k=0}^{K-1}y_{ck}(\theta)f_p\left(y_{sk}(\theta)\right) \tag{9}$$

where

$$\begin{matrix} y_{ck}(\theta) \\ sk \end{matrix} \triangleq \frac{2\sqrt{2S}}{N_0}\int_{kT}^{(k+1)T} y(t)\begin{matrix}\cos\\ \sin\end{matrix}(\omega_c t+\theta)p(t)dt \tag{10}$$

and $f_p(x)$ is the zero memory nonlinearity

$$f_p(x)\triangleq\frac{(1-p)\exp(x)-p\exp(-x)}{(1-p)\exp(x)+p\exp(-x)}=\tanh\left(x-\frac{1}{2}\ln\frac{p}{1-p}\right) \tag{11}$$

The closed loop carrier synchronizer motivated by the above MAP estimation procedure is illustrated in Fig. 2.[7] We herein refer to this loop as the *MAP Estimation Loop for BPSK with unbalanced data*.

In the next section, we specify the tracking performance of this loop in terms of its mean-squared phase error.

TRACKING PERFORMANCE OF THE MAP ESTIMATION LOOP FOR BPSK WITH UNBALANCED DATA

The analysis of the tracking performance (in terms of variance of the phase error) of the loop in Fig. 2 parallels the approach taken in [1] for a similar loop with a hyperbolic tangent nonlinearity. We note, however, that there are some significant differences between the two analyses since several of the steps carried out in [1] depend upon the nonlinearity (e.g., hyperbolic tangent) being an odd function of its argument which is not the case for the nonlinearity $f_p(x)$ (except when $p=1/2$ in which case $f_p(x)=\tanh x$). The complete details of the analysis and these differences are presented in [7] and are omitted here due to lack of space. The results are as follows.

[7] In Fig. 2, the nonlinearity is shown as $f_{\hat{p}}(x)$ since in an actual implementation $\hat{p}$ is an estimate of the true value of p as determined from measurements made on the receiver. Furthermore, in reality, $\hat{p}$ depends on the loop phase error and the detection SNR, R_d. These dependencies are discussed in [7]. For the discussion here, we set $\hat{p}=p$.

Based on a linear loop model (i.e., large loop SNR), the variance of the loop phase error, ϕ, is given by

$$\sigma_\phi^2=\frac{1}{\rho S_L} \tag{12}$$

where $\rho\triangleq S/N_0B_L$ is the loop SNR of an equivalent PLL and analogous to conventional Costas loop terminology, S_L is the "squaring loss" which reflects the penalty paid due to the signal and noise cross-products in the loop error signal and is given by

$$S_L=\frac{\left[(1-p)\overline{f_p\left(2R_d-\sqrt{2R_d}X\right)}^X-p\overline{f_p\left(-2R_d+\sqrt{2R_d}X\right)}^X\right]^2}{(1-p)\overline{f_p^2\left(2R_d-\sqrt{2R_d}X\right)}^X+p\overline{f_p^2\left(-2R_d+\sqrt{2R_d}X\right)}^X} \tag{13}$$

where X is a (0,1) Gaussian random variable and the overbar denotes expectation Note that for $p=1/2$, (13) reduces to

$$S_L=\frac{\left[\overline{\tanh\left(2R_d-\sqrt{2R_d}X\right)}^X\right]^2}{\overline{\tanh^2\left(2R_d-\sqrt{2R_d}X\right)}^X} \tag{14}$$

which is in agreement with Eq. (18) of [2]. Although the statistical averages required in (13) cannot be performed analytically, they are easily evaluated numerically using Gauss-Quadrature techniques.

It is of interest to examine the behavior of the data-reduced carrier synchronization scheme at low SNR (small symbol energy-to-noise ratio) since this is the region where the squaring loss associated with conventional carrier synchronization schemes is large and thus limits their performance. Specifically, the limiting value of (13) for $R_d\to 0$ will also be of interest later on in relation to a more traditional decision feedback scheme derived from ML considerations applied to a received signal with a correlated data sequence. Approximating the in-phase arm nonlinearity of (11) by its expression for small values of its argument, namely,

$$f_p(x)\cong 1-2p+4p(1-p)x \tag{15}$$

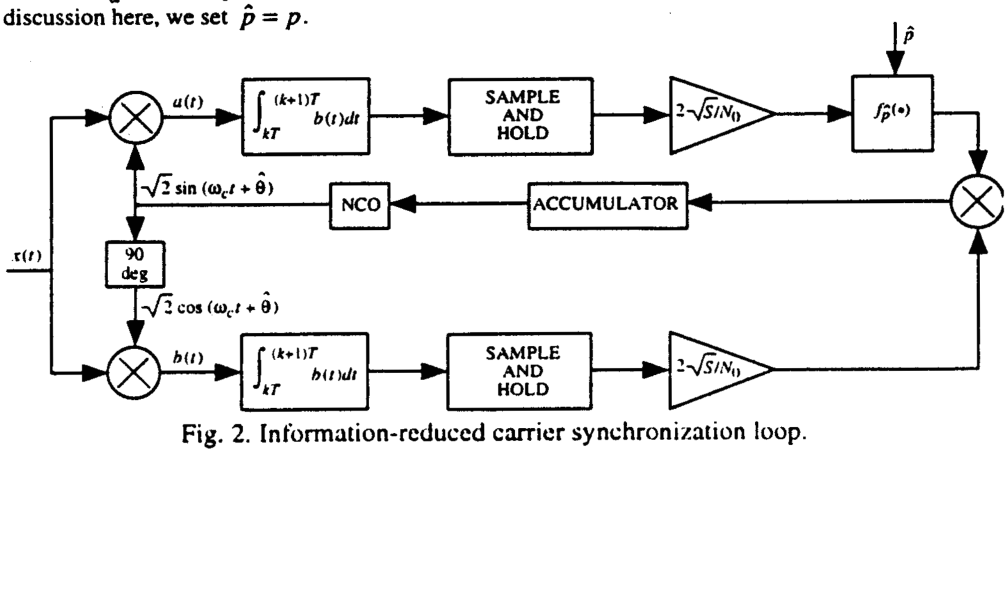

Fig. 2. Information-reduced carrier synchronization loop.

then, it is straightforward to show from (13) that for the information-reduced scheme of Fig. 2

$$\lim_{R_d \to 0} S_L = (1-2p)^2 \qquad (16)$$

Also since S_L is a monotonically increasing function of R_d, then (16) represents a lower bound on the performance of the synchronizer at any SNR.

Fig. 3 is a plot of the squaring loss as determined from the exact expression of (13) versus R_d with p as a parameter. While we recognize that for any given form of data detector p will be a function of R_d, we have chosen to maintain p as a constant parameter in this figure. The reason for this is that the results presented there can then be independent of the specific form of data detector or equivalently the specific form of error correction coding employed, provided that the conditions for an i.i.d and noise dependent error sequence are still met. For any given coding application, only a single point on each curve of constant p would be applicable, namely, the one corresponding to the given error rate behavior of the data detector (decoder) and thus the performance for that particular application would be illustrated by a single curve passing through this series of points. With this interpretation in mind, we note that the curve labelled $p=.5$ corresponds to the in-phase arm nonlinearity $f_p(x)=\tanh x$ and as such agrees with the squaring loss behavior performance of the MAP carrier synchronization structure of uncoded BPSK modulation. For purpose of illustration only, we also include in Fig. 3 curves corresponding to the conventional Costas loop and polarity-type Costas loop for uncoded BPSK. Finally, we note that for improvement to take place in the carrier

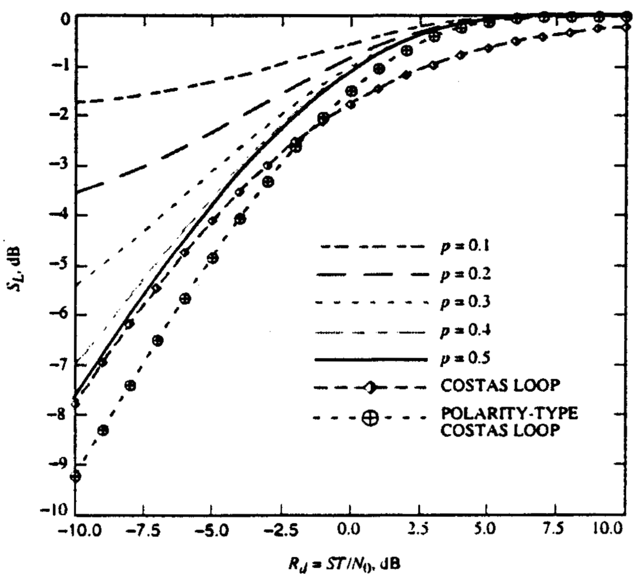

Fig. 3. Squaring-loss performance of an information-reduced carrier synchronization loop; perfect knowledge of p.

synchronizer, the data estimator is not required to operate with symbol error probabilities as small as those typically needed for reliable communication. In fact, a value of $p=.1$, which might be unacceptable for telemetry applications, can result in dramatic improvement in carrier synchronization performance when used in the manner illustrated in Fig. 1.

RELATION TO OTHER STRUCTURES MOTIVATED BY ML CONSIDERATIONS

Suppose that at the outset we had postulated a received signal $y(t)$ consisting of the sum of AWGN and a random binary data source with dependent data symbols, such as that outputted from a convolutional encoder, biphase modulated on a carrier. Based on an observation of $y(t)$ over K symbols and the assumption of a uniformly distributed unknown carrier phase θ, the MAP estimate of this phase is still that value $\hat{\theta}_{MAP}$ that maximizes $p(y(t)|\theta)$. In computing $p(y(t)|\theta)$ for the i.i.d. data source case as in the section on MAP estimation, we first obtained the conditional probability $p(y(t)|\theta,\mathbf{a})$ where $\mathbf{a}$ is the vector of K transmitted data symbols and then averaged over the probability density function (p.d.f.) of $\mathbf{a}$. For an i.i.d. data source as assumed in that section (there the data source actually corresponds to the error sequence), this averaging over the data sequence, i.e., computing the *average-likelihood ratio (ALR)*, can be performed symbol-by-symbol which for binary data results in an LR in the form of a K-fold product of hyperbolic cosine functions. Taking the natural logarithm of this LR (which turns the product into a sum of "ln cosh" functions) and then differentiating with respect to the unknown parameter to find its maximum results in the well-known MAP estimation loop with hyperbolic tangent nonlinearity (derivative of the "ln cosh" function) in its inphase arm and K-symbol accumulator following the I-Q multiplier.

When the data source is not i.i.d. then computation of the ALR by averaging $p(y(t)|\theta,\mathbf{a})$ over $\mathbf{a}$ cannot be performed symbol-by-symbol. Instead the average must be computed over all 2^K possible data sequences which for large K results in an estimation structure with high complexity. Furthermore, since this average cannot be written in the form of a K-fold product, then the advantage obtained by taking the natural logarithm of the ALR before differentiating is now lost [5]. Another possibility for obtaining a carrier phase estimator is to first maximize $p(y(t)|\theta,\mathbf{a})$ over $\mathbf{a}$ resulting in the ML estimate of the data sequence $\hat{\mathbf{a}}_{ML}$ and then substitute this estimate into $p(y(t)|\theta,\mathbf{a})$ before maximizing over the carrier phase θ. Although this *maximum-likelihood ratio (MLR)* approach is suboptimum (since it does not result in maximization of the true LR $p(y(t)|\theta)$ with respect to θ), it nevertheless often produces an estimator whose performance is comparable with that obtained from the ALR approach. A closed loop structure resulting from the MLR approach to carrier phase estimation is suggested in [5] and repeated here in Fig. 4 for clarity of this presentation. We observe that, in principle, the structure requires a maximum-likelihood sequence estimator (MLSE) in its inphase arm prior to multiplication by the appropriate quadrature arm signal. An approximation of this vector decision feedback type of implementation which leads to a simplification in structure can be obtained in, for example, a convolutionally coded system wherein the MLSE is implemented as a Viterbi decoder that outputs symbol-by-symbol decisions after a suitable delay. Using these decisions to multiply the corresponding delayed (by the amount of the decoder delay) symbol-by-symbol outputs of the quadrature arm, one obtains a structure that resembles a polarity-type I-Q Costas loop wherein the matched filter/hard limiter data detector combination in the inphase arm is replaced by a Viterbi decoder. An analysis of the tracking performance of such a structure is straightforward and follows along the lines of that for an uncoded BPSK I-Q loop with hard-limited inphase arm whose squaring loss is given by

$$S_L = \mathrm{erf}^2\sqrt{R_d} \tag{17}$$

Suffice it to say that the analogous result to (17) for the above-mentioned I-Q structure with a Viterbi decoder in its inphase arm is easily shown to be

$$S_L = (1-2p)^2 \tag{18}$$

where p is the symbol error probability performance of the Viterbi decoder under ideal conditions, i.e., with zero phase error. Comparing (18) with (16) and keeping in mind the monotonically increasing nature of S_L with R_d, we conclude that for any $R_d \geq 0$, the information-reduced carrier synchronizer performs at least as well as (and often much better than) the scheme motivated by MLR considerations. The advantage of the MLR scheme, however, is that it can be implemented without the receiver having specific knowledge of p. We hasten to add that all schemes involving coded decision-feedback of one form or another suffer from the potential instability brought about by the decoder delay introduced into the loop.

REFERENCES

[1] M. K. Simon and W. C. Lindsey, "Optimum Performance of Suppressed Carrier Receivers with Costas Loop Tracking," *IEEE Transactions on Communications*, Vol. COM-25, No. 2, February 1977, pp. 215–227.

[2] M. K. Simon, "On the Optimality of the MAP Estimation Loop for Carrier Phase Tracking BPSK and QPSK Signals," *IEEE Transactions on Communications*, Vol. COM-27, No. 1, January 1979, pp. 158–165.

[3] J. J. Stiffler, *Theory of Synchronous Communications*, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1971.

[4] J. H. Chiu and L. S. Lee, "Maximum Likelihood Synchronisers with Arbitrary Prior Symbol Probabilities," *IEEE Proceedings*, Vol. 138, N. 1, February 1991, pp. 50-52.

[5] M. K. Simon, "Closed Loop Carrier Phase Synchronization Techniques Motivated by Likelihood Functions," TDA Progress Report 42-119, July–September 1994, pp. 83–104.

[6] M. K. Simon, H. Tsou, S. Hinedi, and K. Hamdan, "The Performance of a Coherent Residual Carrier PSK System Using Hybrid Carrier Phase Synchronization," *ICC '96 Conference Record*, June 18-22, 1996, Dallas, TX, pp. 1275-1280.

[7] M. K. Simon and V. A. Vilnrotter, "Iterative Information-Reduced Synchronization Using Decision Feedback for Low SNR Applications," TDA Progress Report 42-130, August 15, 1997.

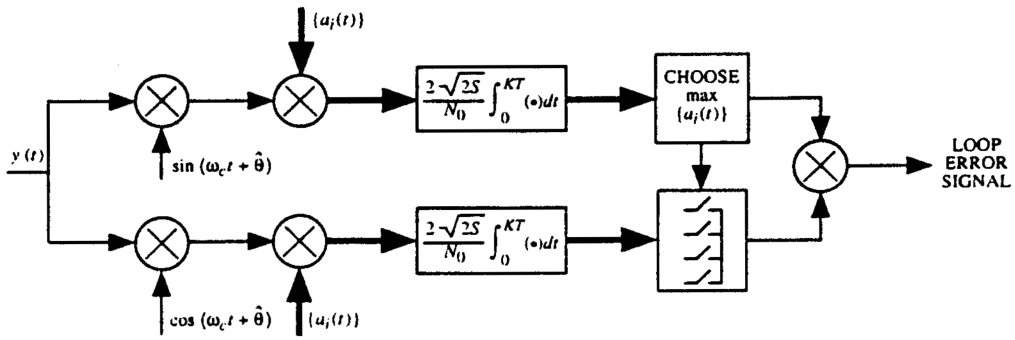

Fig. 4. Closed-loop carrier synchronizer motivated by MLR theory for coded BPSK, where $\{a_i(t)\}$ is a data waveform corresponding to data vector $\mathbf{a}_i$.